AH-1545-CV-19-S

M.Sc. (Final) MATHEMATICS

Term End Examination, 2019-20

INTEGRATION THEORY AND FUNCTIONAL ANALYSIS

Time : Three Hours] **[Maximum Marks : 100**

Note : Attempt any five questions. All questions carry equal marks.

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1. **State and prove Hahn decomposition theorem.**
2. **(a) Define Baire set and prove that every Baire set is $\sigma - finite$.**
   (b) Prove that the union of a sequence of outer regular sets is outer regular.
3. **Show that the linear space R^n of all n-tuples $x = (x_1, x_2, \ldots . x_n)$ of real number is Banach space under the norm**

$$\| x \| = (\sum_{i=1}^{n} |x_i|^2)^{\frac{1}{2}}$$

4. **(a) Let N be a non zero normed linear space and let $s = \{x \epsilon N: \|x\| \leq 1\}$ be a linear subspace of N. prove that N is a Banach space if and only it S is complete.**
   (b) Let T be a linear transformation of a normed linear space N into another normed linear space N' then pjrove that T is continuous if and only it T is continuous at origin.
5. **(a) Let M be a closed linear subspace of a mormed linear space N and ϕ be the natural mapping of N. onto $\frac{N}{M}$ defined by $\phi(x) = x + M$ prove that ϕ is a continuous linear transformation for which $\|\phi\| \leq 1$.**
   (b) Let N be a normed linear space and suppose two norms $\| \|_1$ and $\| \|_2$ are defined on N. Then these norms are equivalent if and only if there exists positive real numbers m and M such that $m\|x\|_1 \leq \|x\|_2 \leq M\|x\|_1$, for every x in N
6. **State and prove closed graph theorem.**
7. **(a) Prove that a closed convex subset c of a Hilbert space H contains a unique vector of smallest norm.**
   (b) Let s be a non empty subset of a Hilbert space H then $S^{\perp}$ is a closed linear subspace of it.
8. **(a) If M is a closed linear subspace of a Hilbert space H then prove that $H = M \oplus M^{\perp}$**
   (b) State and prove Bessel's inequality.
9. **Let T be an operator on a Hilbert space H.**
   Then ∃ a unique operator T^* on H such that
   $(Tx, y) = (x, T^*y) \quad \forall\, x, y \in H$
10. **(a) If T is an operator on a Hilbert space H, then $(Tx, x) = 0, \ \forall x \in H$ if and only if $T = 0$**
    (b) Any arbitrary operator T on a Hilbert space H can be uniquely expressed as $T = T_1 + iT_2$ where T_1 and T_2 are self adjoint operators on H.